PRESS RELEASE

KEK
High Energy Accelerator Research Organization

November 22, 2025
High Energy Accelerator Research Organization

Elucidation of the Frequency Dependence of Displacement Current Density between Capacitor Plates

Dispelling Misconceptions About How Electromagnetic Waves Are Generated.

Executive summary

● Question

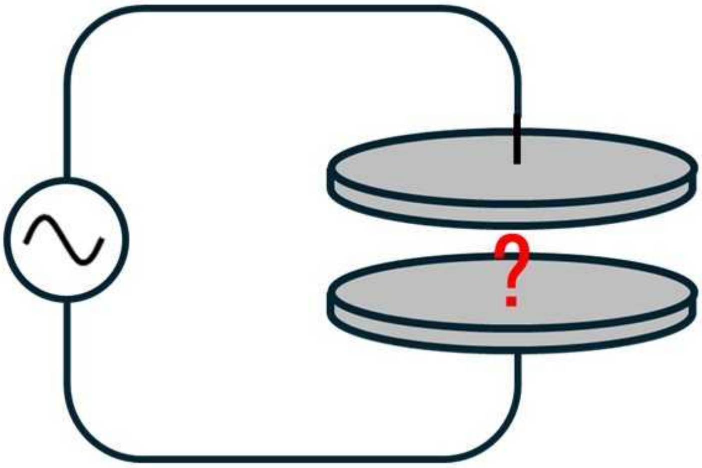
There is a prevailing misconception that the magnetic field between the plates of a capacitor carrying alternating current is generated by the displacement current density existing there, and that this directly leads to the generation of electromagnetic waves. This is incorrect. Then, what about the crucial role of displacement current density in Maxwell's prediction of electromagnetic waves? Apparently a sufficiently coherent and integrated picture of the displacement current density has not yet been available.

● Findings

The frequency (ω) dependence of the Coulombic field and induced field components of the displacement current density between the plates of a parallel-plate capacitor carrying alternating current has been investigated. The results (Figure 1) revealed that the amplitude of the Coulombic field component remains constant regardless of frequency. However, the amplitude of the induced field component is extremely small at low frequencies, increases proportionally to ω^2, and eventually reaches a magnitude comparable to that of the Coulombic field component.

● Meaning

The present paper revieled a fundamental nature of the displacement currenr density that has not been examined since Maxwell's time, providing a coherent and unified view on the displacement current density. It will help avoids unnecessary current and future debates arising from misunderstandings about the phenomena involving dislpacement current density.



Summary

It has been shown that the Coulombic component of the displacement current density between parallel-plate capacitor plates remains constant regardless of frequency, while the induced electric field component is extremely small at low frequencies, increases proportionally to the square of the frequency, and eventually becomes comparable to that of the former. The provided coherent picture will help avoid unnecessary debates about the phenomena involving dislpacement current density.

※

Overview

In Maxwell's equations, the electric field is represented by a single symbol $\boldsymbol{E}$, but it is actually the sum of Coulomb's electric field $\boldsymbol{E}_{\mathrm{C}}$ existing around electric charge and Faraday's induced electric field $\boldsymbol{E}_{\mathrm{I}}$ generated by varying magnetic field,

$$\boldsymbol{E} = \boldsymbol{E}_{\mathrm{C}} + \boldsymbol{E}_{\mathrm{I}}.$$

Everybody knows this, but not much attention has been paid to the fact that the displacement current density, which is the time derivative of $\boldsymbol{E}$ multiplied by the electric permittivity ε_0, also has two components, $\boldsymbol{J}_{\mathrm{C}}$ and $\boldsymbol{J}_{\mathrm{I}}$, each related to each electric field,

$$\boldsymbol{\varepsilon}_0 \frac{\partial \boldsymbol{E}}{\partial \boldsymbol{t}} = \boldsymbol{\varepsilon}_0 \frac{\partial \boldsymbol{E}_{\mathrm{C}}}{\partial \boldsymbol{t}} + \boldsymbol{\varepsilon}_0 \frac{\partial \boldsymbol{E}_{\mathrm{I}}}{\partial \boldsymbol{t}} = \boldsymbol{J}_{\mathrm{C}} + \boldsymbol{J}_{\mathrm{I}}.$$

As a consequence, there exists a prevailing misconception that the magnetic field between the plates of a capacitor carrying alternating current is generated by the displacement current density existing there, and that it directly leads to the generation of electromagnetic waves as the frequency increases. This is incorrect because the displacement current density at low frequencies where the magnetic field is usually measured is dominantly $\boldsymbol{J}_{\mathrm{C}}$ which cannot in principle generate a magnetic field.*) The magnetic field is actually generated directly by the current flowing in the leads to the plates and in the plates. It was also demonstrated that the displacement current density involved in the generation of electromagnetic waves at high frequencies is not $\boldsymbol{J}_{\mathrm{C}}$ but $\boldsymbol{J}_{\mathrm{I}}$.

Then questions may arise: what about $\boldsymbol{J}_{\mathrm{I}}$ in the low-frequency region, and what about $\boldsymbol{J}_{\mathrm{C}}$ in the high-frequency region? Apparently a sufficiently coherent and integrated picture of the displacement current density is not yet available. The present paper has addressed this issue.

It has been investigated how the amplitude J_{C0} of $\boldsymbol{J}_{\mathrm{C}}$ and the amplitude of J_{I0} of $\boldsymbol{J}_{\mathrm{I}}$ in the space between the plates of a parallel-plate capacitor vary with angular frequency ω when an alternating current, $I_0 \sin \omega t$ flows thouth it. We assume that the plates are circular disks of radius r and the distance between the plates is sufficiently small so that the edge effects of the plates can be neglected.

The results reveal that when the amplitude I_0 of the current is constant (or normalized to I_0), J_{C0} remains constant regardless of the frequency ω. On the other hand, the amplitude J_{I0} is extremely small at low frequencies, increases proportionally to ω^2, slows its increase rate around $\omega \sim c/r$, and eventually reaches a magnitude comparable to J_{C0}. Here, c is the speed of light, and c/r is the reciprocal of the time that light takes to travel a distance r. As the plate radius r decreases, the frequency $\omega \sim c/r$ increases further, and J_{I0} becomes even smaller.

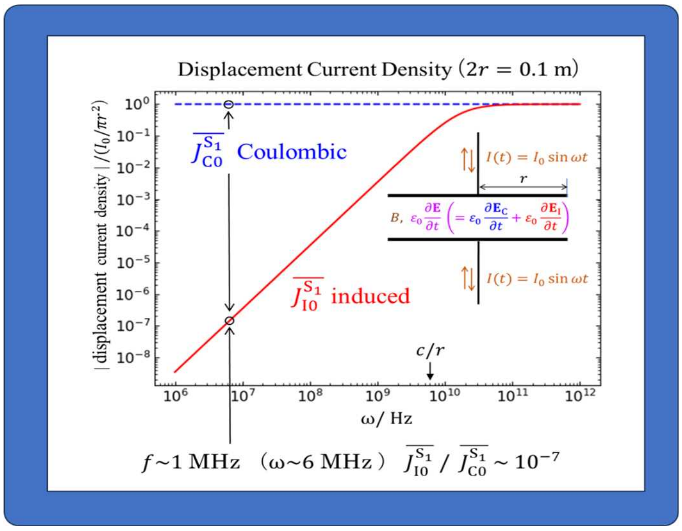


Figure 1. Frequency dependence of the amplitudes of the components of the displacement current density spatially averaged over a flat surface in the middle of the capacitor plates.

Figure 1 shows the frequency dependence of $J_{\mathrm{C}0}$ and $J_{\mathrm{I}0}$ for the case of $r = 5$ cm (diameter 10 cm) as a representative size for parallel-plate capacitors where magnetic field measurements are usually performed. While $J_{\mathrm{C}0}$ is spatially uniform in the area, $J_{\mathrm{I}0}$ is not. Therefore, the actual comparison is made between their spatial average over a flat surface of radius r in the middle of the capacitor plates.

It is seen that even at quite high AC frequencies in everyday term like $f \sim 1$ MHz ($\omega = 2\pi f \sim 6$ MHz), the amplitude $J_{\mathrm{I}0}$ is still only 10^{-7} of the amplitude $J_{\mathrm{C}0}$. This confirms the argument is valid that the displacement current density at frequencies where magnetic fields are measured corresponds to the $\boldsymbol{J}_{\mathrm{C}}$, which does not generate a magnetic field. As ω increases, $\boldsymbol{J}_{\mathrm{I}}$—the source of electromagnetic waves—grows larger. This makes the intensity of the electromagnetic waves increase and gradually become detectable. (The frequency at which detection becomes possible is rather a matter of the detector's sensitivity.) It is also shown that $\boldsymbol{J}_{\mathrm{C}}$, which is not the source of either the magnetic field between the plates or the electromagnetic waves, always exists with its amplitude $J_{\mathrm{C}0}$ remaining constant and is always larger than or comparable to $J_{\mathrm{I}0}$.

The present paper has demonstrated a fundamental property of the displacement current density that has not been sufficiently examined since Maxwell's time. The coherent picture provided helps avoid current and future unnecessary debates involving displacement current density. In the Editor's note for the paper published in the American Journal of Physics, it was evaluated as: "This would make an appropriate supplementary topic for advanced-level electromagnetism lectures." It is also expected to be added to textbooks.

*) T. Hyodo, "Maxwell's displacement current and the magnetic field between capacitor electrodes," *Eur. J. Phys.* **43**(6), 065202 (2022).

Research group

Slow Positron Facility, Institute of Materials Structure Science, High Energy Accelerator Research Organization (KEK)

Associate Researcher HYODO, Toshio

Paper Information

Toshio Hyodo, "Frequency dependence of the displacement current density", American Journal of Physics, 93(11), 891-896 (2025).

DOI: https://doi.org/10.1119/5.0237905

Editor’s notes: https://pubs.aip.org/aapt/ajp/issue/93/11

Contact Information

<Author Contact>

Toshio Hyodo

Email: hyodot@post.kek.jp

<Press Contact>

High Energy Accelerator Research Organization (KEK) Public Relations Office

Tel: 029-879-6047

e-mail: press@kek.jp